Model Solutions

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Thursday 16 May 2019

Afternoon | Paper Reference **8FM0-21**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
21: Further Pure Mathematics 1
(Part of options A, B, C and D)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

1. (a) Write down the t-formula for $\sin x$.

(1)

(b) Use the answer to part (a)

(i) to find the exact value of $\sin x$ when

$$\tan\left(\frac{x}{2}\right)=\sqrt{2}$$

(ii) to show that

$$\cos x = \frac{1-t^2}{1+t^2}$$

(4)

(c) Use the t-formulae to solve for $0 < \theta \leqslant 360°$

$$7\sin\theta + 9\cos\theta + 3 = 0$$

giving your answers to one decimal place.

(4)

a) $\sin x = \dfrac{2t}{1+t^2}$

bi) let $\tan\frac{x}{2} = t$

$\Rightarrow t = \sqrt{2}$

$\therefore$ Using equation $\sin x = \frac{2t}{1+t^2}$:

$\sin x = \dfrac{2\sqrt{2}}{1+(\sqrt{2})^2} = \dfrac{2\sqrt{2}}{1+2}$

$\sin x = \dfrac{2\sqrt{2}}{3}$

bii) Method 1

$\cos x = \dfrac{\sin x}{\tan x}$

Since $\sin x = \dfrac{2t}{1+t^2}$

and $\tan x = \dfrac{2t}{1-t^2}$:

$\dfrac{\;\frac{2t}{1+t^2}\;}{\frac{2t}{1-t^2}} = \dfrac{1-t^2}{1+t^2} = \cos x$

(as required)

Method 2

$\cos^2 x = 1 - \sin^2 x$

Since $\sin x = \dfrac{2t}{1+t^2} \Rightarrow \sin^2 x = \dfrac{4t^2}{(1+t^2)^2}$

$\cos^2 x = 1 - \dfrac{4t^2}{(1+t^2)^2}$

$= \dfrac{(1+t^2)^2}{(1+t^2)^2} - \dfrac{4t^2}{(1+t^2)^2} = \dfrac{t^4+2t^2-4t^2+1}{(1+t^2)^2}$

$= \dfrac{(1-t^2)^2}{(1+t^2)^2}$

$\therefore \cos x = \dfrac{1-t^2}{1+t^2}$ (as required)

Question 1 continued

c) Since $\sin\theta = \frac{2t}{1+t^2}$ and $\cos\theta = \frac{1-t^2}{1+t^2}$

$$7\sin\theta + 9\cos\theta + 3 = 7\left(\frac{2t}{1+t^2}\right) + 9\left(\frac{1-t^2}{1+t^2}\right) + 3 = 0$$

$$\frac{14t}{1+t^2} + \frac{9-9t^2}{1+t^2} + \frac{3(1+t^2)}{1+t^2} = 0$$ (multiply both sides by $(1+t^2)$)

$$14t + 9 - 9t^2 + 3 + 3t^2 = 0$$

$$6t^2 - 14t - 12 = 0$$ (divide both sides by 2)

$$3t^2 - 7t - 6 = 0$$

$$(t-3)(3t+2) = 0$$

$t = 3$ or $t = -\frac{2}{3}$

As $t = \tan\frac{\theta}{2}$

$\tan\frac{\theta}{2} = 3$ $\qquad$ $\tan\frac{\theta}{2} = -\frac{2}{3}$

$\frac{\theta}{2} = 71.6^\circ$ (1dp) $\qquad$ $\frac{\theta}{2} = -33.7^\circ, 146.3^\circ$ (1dp)

$\theta = 143.1^\circ$ (1dp) $\qquad$ $\theta = 292.6^\circ$ (1dp)

(Total for Question 1 is 9 marks)

2. A student was set the following problem.

Use algebra to find the set of values of x for which

$$\frac{x}{x-24} > \frac{1}{x+11}$$

The student's attempt at a solution is written below.

1) $x(x-24)(x+11)^2 > (x+11)(x-24)^2$

2) $x(x-24)(x+11)^2 - (x+11)(x-24)^2 > 0$

3) $(x-24)(x+11)\left[x(x+11) - x - 24\right] > 0$ Line 3

4) $(x-24)(x+11)\left[x^2 + 10x - 24\right] > 0$

5) $(x-24)(x+11)(x+12)(x-2) > 0$

6) $x = 24,\ x = -11,\ x = -12,\ x = 2$

7) $\{x \in \mathbb{R} : -12 < x < -11\} \cup \{x \in \mathbb{R} : 2 < x < 24\}$ Line 7

There are errors in the student's solution.

(a) Identify the error made

(i) in line 3

(ii) in line 7

(2)

(b) Find a correct solution to this problem.

(4)

ai) Bracket error → '-24' should be '24' in the square brackets.

aii) Should be $\{x \in R : x < -12 \ \cup\ -11 < x < 2 \ \cup\ x > 24\}$

⇒ they have reversed the inequality.

b) $(x-24)(x+11)\left[x(x+11) - (x-24)\right] > 0$

$(x-24)(x+11)\left[x^2 + 11x - x + 24\right] > 0$

$(x-24)(x+11)(x^2 + 10x + 24) > 0$

$(x-24)(x+11)(x+4)(x+6) > 0$

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Question 2 continued

b continued) Critical Values of x: $-11, -6, -4, 24$

$\therefore \{x \in \mathbb{R} : x < -11\} \cup \{x \in \mathbb{R} : -6 < x < -4\} \cup \{x \in \mathbb{R} : x > 24\}$

(Total for Question 2 is 6 marks)

3. Julie decides to start a business breeding rabbits to sell as pets.

Initially she buys 20 rabbits. After t years the number of rabbits, R, is modelled by the differential equation

$$\frac{dR}{dt} = 2R + 4\sin t \qquad t > 0$$

Julie needs to have at least 40 rabbits before she can start to sell them.

Use two iterations of the approximation formula

$$\left(\frac{dy}{dx}\right)_n \approx \frac{y_{n+1} - y_n}{h}$$

to find out if, according to the model, Julie will be able to start selling rabbits after 4 months.

(7)

Population after 4 months is required over two iterations :

$h = \frac{1}{6}$

when $t_0 = 0$, $R_0 = 20$:

$\left(\frac{dR}{dt}\right)_0 = 2(20) + 4\sin(0)$

$= 40$

$\Rightarrow \frac{dy}{dx} = 40 = \frac{R_1 - 20}{\frac{1}{6}}$

$\frac{20}{3} = R_1 - 20$

$R_1 = \frac{80}{3}$

when $t = h = \frac{1}{6}$, $R = R_1 = \frac{80}{3}$

$\left(\frac{dR}{dh}\right)_1 = 2\left(\frac{80}{3}\right) + 4\sin\left(\frac{1}{6}\right)$

$= 53.9969\ldots$

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Question 3 continued

$R_2 = R_1 + h\left(\frac{dh}{dt}\right)_1 = \frac{80}{3} + \frac{1}{6}(53.9969...)$

$= 35.666...$ rabbits

$\Rightarrow$ 35 or 36 robbits

As 35 and 36 < 40, Julie will not be able to sell her rabbits after 4 months.

(Total for Question 3 is 7 marks)

4.

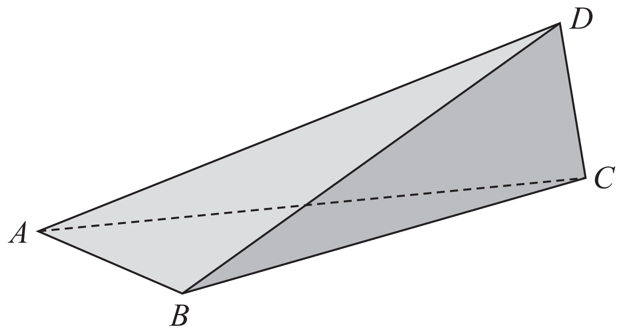


Figure 1

Figure 1 shows a sketch of a solid doorstop made of wood. The doorstop is modelled as a tetrahedron.

Relative to a fixed origin O, the vertices of the tetrahedron are A (2, 1, 4), B (6, 1, 2), C (4, 10, 3) and D (5, 8, d), where d is a positive constant and the units are in centimetres.

(a) Find the area of the triangle ABC.

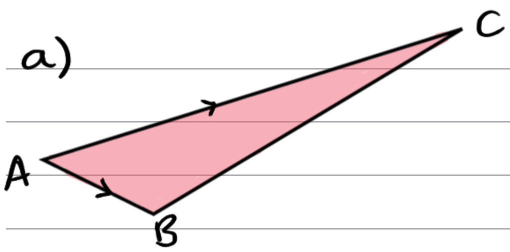
(4)

Given that the volume of the doorstop is 21 cm^3

(b) find the value of the constant d.

(4)

a)



$\vec{AB} = \begin{pmatrix}6\\1\\2\end{pmatrix} - \begin{pmatrix}2\\1\\4\end{pmatrix} = \begin{pmatrix}4\\0\\-2\end{pmatrix}$

$\vec{AC} = \begin{pmatrix}4\\10\\3\end{pmatrix} - \begin{pmatrix}2\\1\\4\end{pmatrix} = \begin{pmatrix}2\\9\\-1\end{pmatrix}$

Using the cross product:

$\begin{vmatrix} i & j & k \\ 4 & 0 & -2 \\ 2 & 9 & -1 \end{vmatrix} = i(-(-18)) - j(-4-(-4)) + k(36-0)$

$= 18i + 0j + 36k$

Area $ABC = \sqrt{18^2 + 36^2}$

$= \boxed{9\sqrt{5} \text{ cm}^2}$

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Question 4 continued

b) Volume $ABCD = 21$

$$\vec{AD} = \begin{pmatrix} 5 \\ 8 \\ d \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \\ d-4 \end{pmatrix}$$

Using the dot product:

$\vec{AD} \cdot (\text{Area of } ABC)$

$$= \begin{pmatrix} 3 \\ 7 \\ d-4 \end{pmatrix} \cdot \begin{pmatrix} 18 \\ 0 \\ 36 \end{pmatrix} = 54 + 36d - 144$$

$$\Rightarrow \frac{1}{6}(36d - 90) = 21$$

$$36d - 90 = 126$$

$$36d = 216$$

$$d = 6$$

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

5.

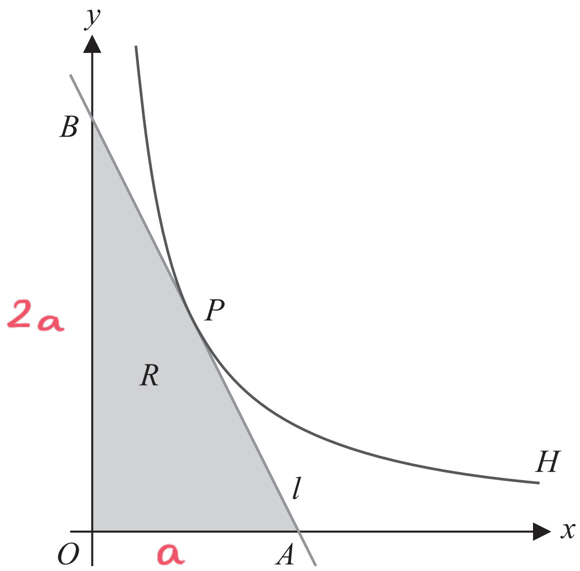

Figure 2

Figure 2 shows a sketch of part of the rectangular hyperbola H with equation

$$xy = c^2 \qquad x > 0$$

where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$ lies on H.

The line l is the tangent to H at the point P.

The line l crosses the x-axis at the point A and crosses the y-axis at the point B.

The region R, shown shaded in Figure 2, is bounded by the x-axis, the y-axis and the line l.

Given that the length OB is twice the length of OA, where O is the origin, and that the area of R is 32, find the exact coordinates of the point P.

(10)

Area of $R = \frac{2a \times a}{2} = a^2 = 32$

$a = 4\sqrt{2}$

$2a = 8\sqrt{2}$

Working out equation for line l:

$\frac{y_1 - y_2}{x_1 - x_2} = \frac{8\sqrt{2} - 0}{0 - 4\sqrt{2}} = -2$

$y = -2x + 8\sqrt{2}$

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Question 5 continued

Point P is when $xy = c^2$ and $y = -2x + 8\sqrt{2}$ meet

Sub ℓ into H:

$x(-2x + 8\sqrt{2}) = c^2$

$-2x^2 + 8x\sqrt{2} = c^2 \quad — (1)$

ℓ is a tangent of H, so differentiate H:

$y = \frac{c^2}{x} = c^2x^{-1}$

$\frac{dy}{dx} = -c^2x^{-2} = \frac{-c^2}{x^2}$

when at P, $\frac{dy}{dx} = -2$

$-2 = -\frac{c^2}{x^2}$

$2x^2 = c^2 \quad — (2)$

Sub (2) into (1):

$-2x^2 + 8x\sqrt{2} = 2x^2$

$4x^2 - 8x\sqrt{2} = 0$ (divide both sides by 2)

$x(2x - 4\sqrt{2}) = 0$

$2x - 4\sqrt{2} = 0$

$2x = 4\sqrt{2}$

$x = 2\sqrt{2}$

$\therefore y = -2(2\sqrt{2}) + 8\sqrt{2}$

$y = 4\sqrt{2}$

$\therefore P(2\sqrt{2}, 4\sqrt{2})$

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 1 IS 40 MARKS